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Angular Momentum Control in Nonlinear Flight

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A rational approach to control angular motion of aircraft in highly nonlinear flight regimes is presented. The method seeks to change the angular velocity by directing the aerodynamic moment so that it is most effective in achieving this objective. Validation of the method is presented by simulating two spins of a light aircraft. The approach is shown to be successful in stopping the rotation for effecting spin recovery.

Nomenclature

$\boldsymbol{G}$	= moment vector
$\boldsymbol{H}$	= angular momentum vector
I	= moment of inertia
L, M, N	= moment vector components in body axes
P	= roll rate
Q	= pitch rate
R	= yaw rate
$\boldsymbol{u}$	= control vector
V	= airspeed
α	= angle of attack
β	= sideslip angle
δ_A	= aileron deflection
δ_E	= elevator deflection
δ_R	= rudder deflection
$\boldsymbol{\omega}$	= angular velocity vector

Introduction

SINCE the early days of aviation, inadvertent spin entry has been documented in accident statistics as a serious cause of aviation fatalities.[1,2] As an illustration, the accident statistics published by the National Transportation Safety Board (NTSB) from 1964 to 1994 show that stall/spin-related accidents were among the leading causes of accidents and fatalities in general aviation. For example, from 1965 to 1973, 2% of all accidents were caused by spins, but over the same period 12% of the total fatalities were attributed to spin accidents. In particular, the NTSB report, covering the 1967–1969 period, indicates that 27% of all accidents were attributed to stall/spin. As of 1982, NTSB started to report stall/spin-related accident statistics under a different category, loss of control in flight, to include detailed categories such as stall, stall/spin, and stall/mush. Statistical data for 13 years from 1982 to 1994 show that 13% of all accidents were attributed to loss of control in flight. Five percent of all accidents (21% of all fatal accidents) were due to loss of control. From spin accident statistics, one can infer that loss of control related to stall/spin is still a major cause of fatalities. Accidents due to loss of control is discouraging because a boom in general aviation is expected in the near future. The rates of fatal and total accidents caused by loss of control in flight remain steady during this period. This implies that more hours flown will result in more casualties, including stall/spin accidents.

Such spins happen mostly to unsuspecting pilots, most likely those holding only a private license, as they are not required by law to be trained to recover from spins. At low altitudes, such as during departure and approach, the associated low airspeeds exacerbate the problem. The present work seeks to provide a means of avoiding the spin or recovering from it, so as to reduce its contribution to accidents.

During the 1970s and 1980s, considerable stall/spin research was conducted by the NASA Langley Research Center. Most of the work led to determining airplane configurations and mass distributions to make the airplane "spin resistant" or easy to recover from a fully developed spin.[3–8] These efforts involved extensive wind tunnel and flight testing. Research based on automatic recovery is scarce, however. One such endeavor, using an optimal approach for recovery, can be found in Ref. 9. In this work, a function optimization was attempted to recover from a flat spin by minimizing time derivatives of angle of attack, sideslip angle, and angular rates. Recent research proposed recovery from nonlinear flight conditions (the falling leaf maneuver was used) by the application of a moment along the angular momentum axis.[10] In this study, a damping moment is applied to suppress the angular momentum vector. Unlike the spin, the motion shows strong all-axes coupling, such as in-phase roll and yaw rate, with rapid angle of attack and sideslip angle traverse. This study presents a new concept for the direct suppression of the angular momentum vector and shows it to be a successful means of arresting the airplane's rotation.

This paper extends this concept to present a scheme for angular momentum control under highly complex and nonlinear flight conditions, specifically applied to enable recovery from maneuvers, entered intentionally or inadvertently, in which large angles of attack and/or sideslip and large angular rates exist. Spins are used to illustrate the concept. The basis for the approach is that, to change the angular velocity, Newton's Second Law dictates that the corresponding angular momentum vector is what should be modified. This scheme is based on the notion that the necessary control inputs are difficult to determine when angular rates are present, for the following reasons:

1) It is difficult for the pilot to perceive correctly the orientation of the angular velocity vector, or axis of rotation, relative to the body axes. This could be overcome using measurements.

2) The angular velocity vector, if known, is not the axis about which the control moment should be applied, unless the body axes are principal axes with equal principal moments of inertia. The control moment should be applied about the angular momentum vector.

3) If the angular momentum axis is not used, gyroscopic effects will cause motions that are difficult to anticipate.

4) The standard controls—rudder, aileron, and to a lesser extent, elevator and thrust—produce moments about more than one axis, further complicating the determination of the required controls.

5) When large angles of attack and sideslip are present, the control surface effectiveness is usually reduced, in a manner dependent on the local angle of attack at the control surface, which in turn depends on the angular velocity vector.

In flight conditions involving low rotation rates (even with large angles of attack) some of these reasons are no longer strong contributing factors. With considerable training in a particular aircraft,

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pilots develop a "feel" or strategy for the controls required to recover from maneuvers such as the spin or the falling leaf. The strategy focuses on spin recovery, and depends on the inertia properties (the inertia tensor) of the aircraft and the aerodynamic characteristics at the equilibrium angle of attack. The training approach works well as long as no unanticipated equilibrium states occur. If this happens, the results could be disastrous.

The objective of this work is to present a rational and robust scheme to recover from such maneuvers. The intent is either to provide the pilot with an advisory as to what control input sequence is required, or to be implemented as an automatic controller that will produce these controls. The scheme is easier to visualize as a device which arrests a rotation i.e., a regulator—when rotation is stopped or considerably reduced, control of the aircraft will be much easier. The recovery of the angles of attack and sideslip, the airspeed, and the attitude angles are not taken into account in this work. The paper focuses on a method to control angular momentum to achieve this objective.

Aerodynamic and Control Forces and Moments Modeling

A recent effort to model aerodynamic forces (and moments) during high angle-of-attack and large angular rate maneuvers, such as spins, was based on the application of the multipoint model.[11,12] In this approach, the aerodynamic forces are estimated as a distribution over each aircraft surface component (wing, tail, fuselage, etc.). The coefficients of these distributions are estimated by regression, using available spin flight test data.[13] That effort was successful in that it gave a good reproduction of the forces and moments.

It is useful to separate the aerodynamic forces and moments into two parts: 1) state dependent, and 2) control dependent.

The first part depends strictly on the components of the relative wind velocity in body axes (or equivalently, the airspeed and the angles of attack and sideslip) as well as the angular velocity. In this approach, the aerodynamic forces are estimated as a distribution over each aircraft surface component, excluding control surface components.

The second part depends on the control deflections, which are arbitrary inputs. The strategy to model the control dependent part of the forces and moments is to find the control derivatives of moment as a function of local angle of attack. The estimation of a control derivative as a function of the angle of attack is done by curve fitting data obtained from existing static wind tunnel tests.[14] This is then subtracted from the total to obtain the state-dependent part, and regression is used to calculate the associated parameters. The model for the aircraft used to validate this approach is discussed in detail in Ref. 15. The resulting model is highly nonlinear.

Angular Momentum Suppression

As shown in Fig. 1, the angular momentum and angular velocity vectors of the aircraft are different in direction because the moments of inertia are not always principal and seldom equal. To modify the angular velocity, the corresponding angular momentum must be modified by applying aerodynamic moments. To achieve this, the aerodynamic moment must be applied in the direction of the desired change in angular momentum. The example presented here is that of recovery from a spin. In this case, the aerodynamic moment vector is preferably parallel to and opposite the angular momentum vector, so that the angular velocity is reduced eventually to zero. Because the state-dependent moments are not controllable, and because the control-dependent moments are usually weakened in a fully developed spin, this objective may not be achievable. The objective of this work is to maximize the component of the total moment suppressing the angular momentum at any instant.

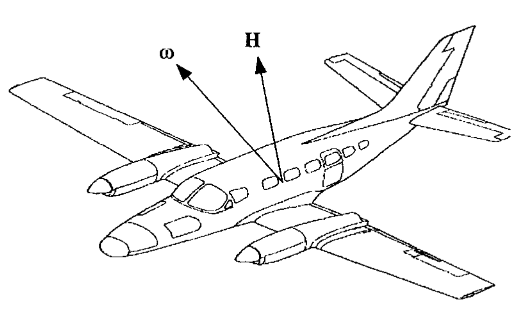

Fig. 1 Angular velocity and momentum vectors of an aircraft.

Methodology

The angular momentum is $\boldsymbol{H} = [I]\boldsymbol{\omega}$, where $[I]$ is the inertia tensor. The aerodynamic moment $\boldsymbol{G}$ is the vector sum of the state-dependent moment $\boldsymbol{G}_S$ and the control dependent one, $\boldsymbol{G}_C$. Then

$$\boldsymbol{G}_C = \boldsymbol{G} - \boldsymbol{G}_S$$

We desire to have $\boldsymbol{G}$ acting in the opposite direction of $\boldsymbol{H}$, so that we can achieve the appropriate rate of change in angular momentum that will stop the rotation.

Let $\boldsymbol{G} = -K\boldsymbol{H}$, where K is a gain to be determined by testing the response of the system, in the same fashion as the gain is fine-tuned in a classic single input/single output feedback design problem. Therefore,

$$\boldsymbol{G}_C = -K\boldsymbol{H} - \boldsymbol{G}_S, \qquad K > 0$$

The right-hand side depends on the state $\boldsymbol{V}$ and ω and the inertia tensor. Knowing the desired value of $\boldsymbol{G}_C$, the control deflections can be obtained by an inverse solution. A straightforward way to do this is by finding the control deflections vector $\boldsymbol{u} = \{\delta_A, \delta_E, \delta_R\}$ that will minimize the difference between the available control moment $\boldsymbol{G}_C(\boldsymbol{u})$ and its desired value $\boldsymbol{G}_C$.

If we define a cost function

$$J = [\boldsymbol{G}_C - \boldsymbol{G}_C(\boldsymbol{u})]^T[\boldsymbol{G}_C - \boldsymbol{G}_C(\boldsymbol{u})]$$

then the problem is to find $\boldsymbol{u}$ for minimum J. This scheme is illustrated in the diagram in Fig. 2.

To simplify the computational algorithm, we choose

$$\begin{aligned} J &= [\boldsymbol{G} + K\boldsymbol{H}]^T[\boldsymbol{G} + K\boldsymbol{H}] \\ &= [\boldsymbol{G}_S + \boldsymbol{G}_C + K\boldsymbol{H}]^T[\boldsymbol{G}_S + \boldsymbol{G}_C + K\boldsymbol{H}] \\ &= (L_S + L_C + KPI_X)^2 + (M_S + M_C + KQI_Y)^2 \\ &\quad + (N_S + N_C + KRI_Z)^2 \end{aligned}$$

and find $\boldsymbol{u}$ to minimize J (its desired value being zero), given that $\boldsymbol{G}_C$ is a function of $\boldsymbol{u}$.

The controls have lower and upper bounds, which are taken as constraints in the minimization scheme. Those bounds are taken to be the same as the actual deflection limits for the aircraft used in this paper. The optimization package NPSOL (Nonlinear Programming SOLution[16]) is used to solve the preceding constrained optimization problem. This procedure is repeated 10 times per second, using the current value of the state obtained from the previous steps.

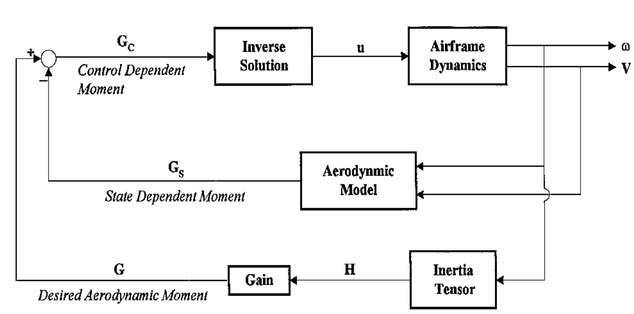

Fig. 2 Diagrammatic representation of the angular momentum control augmentation system.

The simulation performed in this work uses a single-time step for each iteration using the fourth-order Runge–Kutta algorithm, and calls NPSOL to compute the controls for the next time step. The rigid-body, six-degrees-of-freedom, nonlinear Euler equations of motion with quaternions are used.

Results

In this section, the preceding recovery algorithm is applied to demonstrate that it generates time histories for the controls that are successful in arresting the aircraft rotation. To allow for smoother behavior of the aileron, a first-order lag (servo) is included. Its equation is:

$$\dot{\delta}_A = \tau \delta_A + \delta_{A_C}$$

where δ_{A_C} is the computed or command aileron deflection and τ is the servo time constant.

The flight test measurements of Ref. 13 are used for comparison. Specifically, two spins, referred to herein as spins G and P are selected. Spin G is the "right spin of six turns with baseline configuration at idle power with aileron neutral," on page 72 (see Ref. 13). It is a moderately steep spin. Spin P is described as "flat spin of baseline configuration at idle power with aileron neutral," on page 88 (see Ref. 13). It is a much flatter spin than G, where the pilot deployed the spin chute at $t = 52$ s, indicating it may have been a difficult spin to recover from.

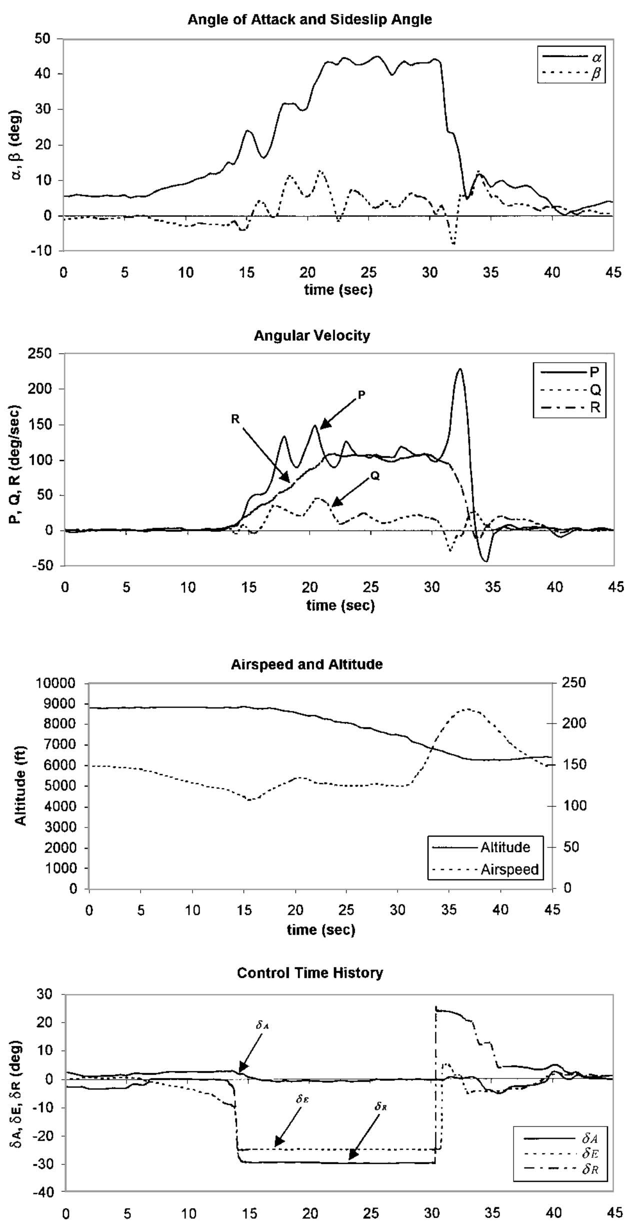

Fig. 3 Measured time history for spin G.

Spin G

Figure 3 shows the flight test control deflections and state time histories from Ref. 13. In Fig. 4, the state time histories are repeated from flight test up to $t = 30$ s. Thereafter, the flight test control inputs by the pilot are used in the simulation, using for the initial state the values at $t = 30$ to obtain the state past 30 s. This is intended to illustrate the validity of the aerodynamic model used. A comparison with the plots in Fig. 3 verifies this.

The scheme is used to produce the control inputs required for reducing the angular velocity and the results are given in Fig. 5. Also shown are trajectories with the controls set in their neutral positions. The dotted lines are the trajectories when, at $t = 30$ s, the controls are neutralized, i.e.,

$$\boldsymbol{u} = \boldsymbol{0}, \qquad t \geq 30 \text{ s}$$

whereas the solid lines show the results of the present recovery algorithm computation, which was also initiated at $t = 30$ s. The gain K here is $2.2|\omega|$. Because this is a steep spin, the neutral controls lead to a recovery, but the computed controls allow a faster and more complete recovery.

The gain varies linearly with the magnitude of the angular velocity. This helps in improving the behavior when the recovery nears completion and was determined to improve the system performance.

The recovery trajectories are similar to those of the flight test, all angular rates returning essentially to zero at $t = 45$ s, 15 s from the start of the recovery. The exception is the roll rate, which increased

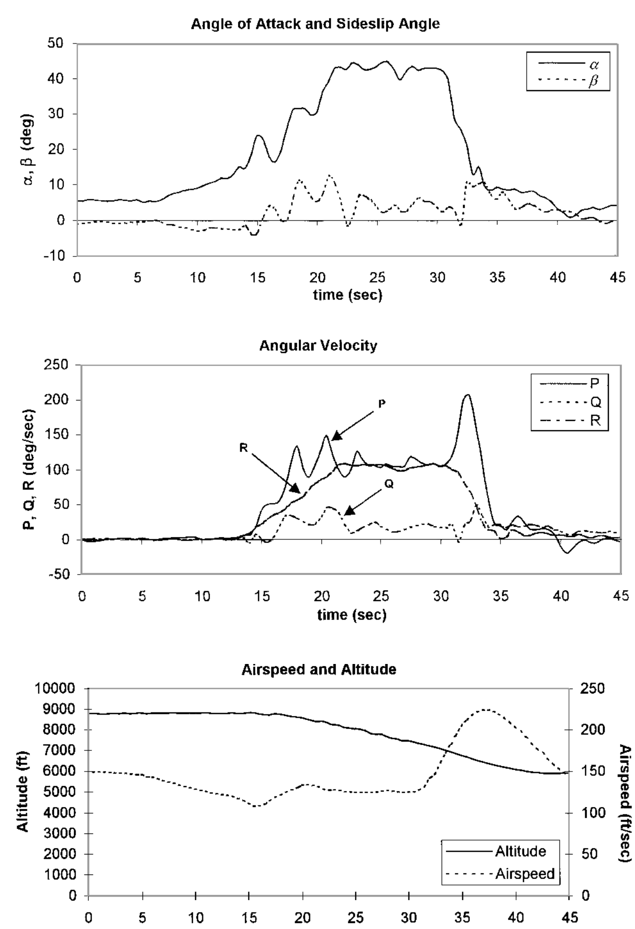

Fig. 4 Spin recovery simulation with measured control (simulation start at 30 s).

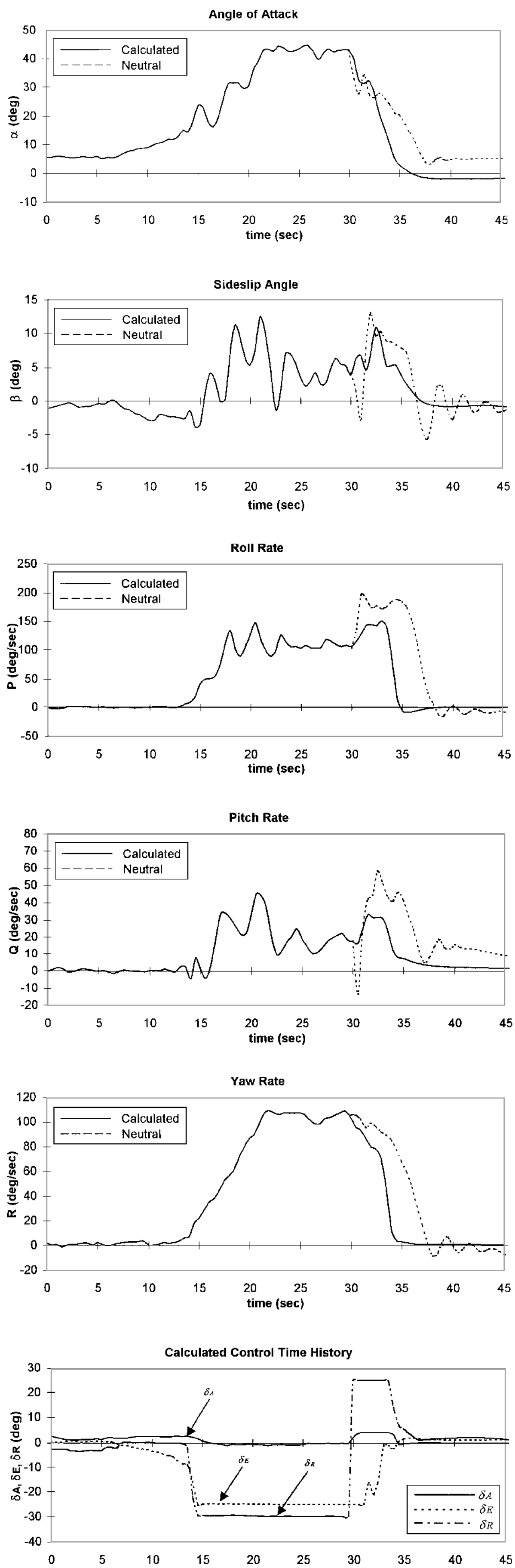

Fig. 5 Automatic recovery (——) and neutral controls (initiated at 30 s).

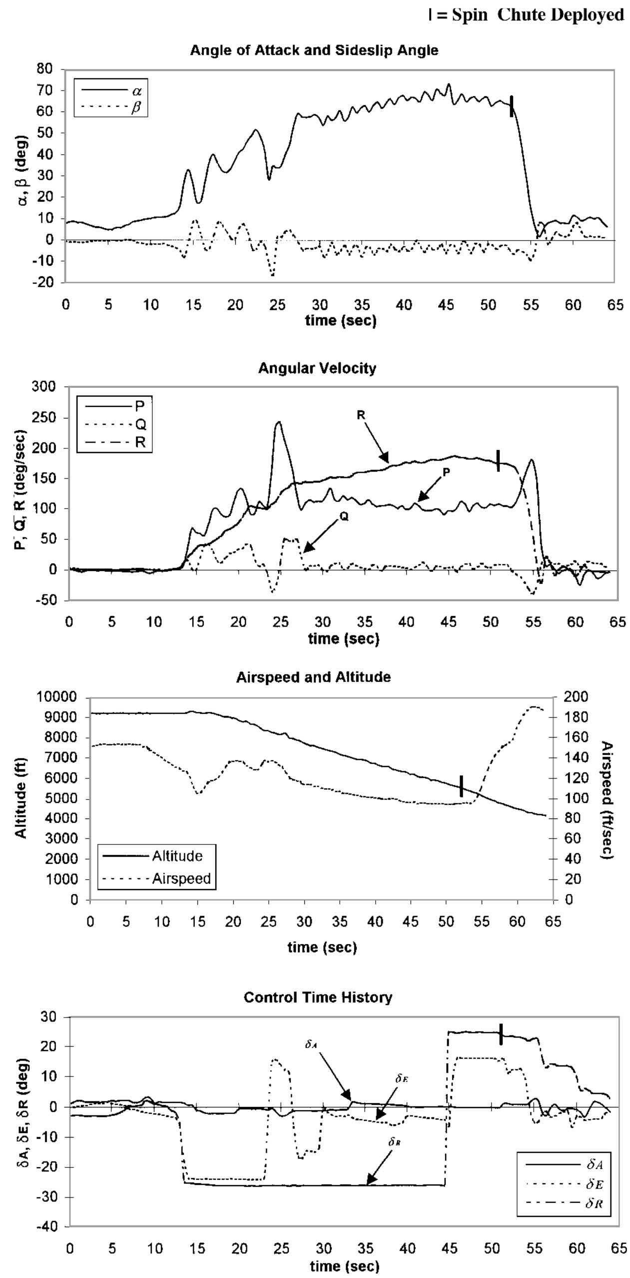

Fig. 6 Measured time history for spin P.

in the flight test before it was restored because the aileron deflection was kept nearly neutral. With the appropriate controls obtained from the current algorithm, this spike was avoided.

Spin P

The data are presented for this spin in the same sequence as for spin G. Figure 6 shows the flight test data. The point at which the spin chute was deployed is marked by a vertical line segment at $t = 52$ s. Figure 7 shows the computed trajectories where the dashed line corresponds to neutralized controls, starting from $t = 40$ s, and the solid lines are for the controls calculated according to the current scheme. Unlike spin G, the neutralized control inputs did not help in this case. The angles of attack and sideslip and the angular rates maintain a large nonzero value, indicating a stable equilibrium spin that may be difficult to recover from. The algorithm did manage to produce the control deflections necessary to stop the roll and yaw rates and considerably reduce the pitch rate in 15 s.

Effect of the Gain K

When the value of K is increased, some controls oscillations take place, but when decreased, the recovery is slower. This behavior is

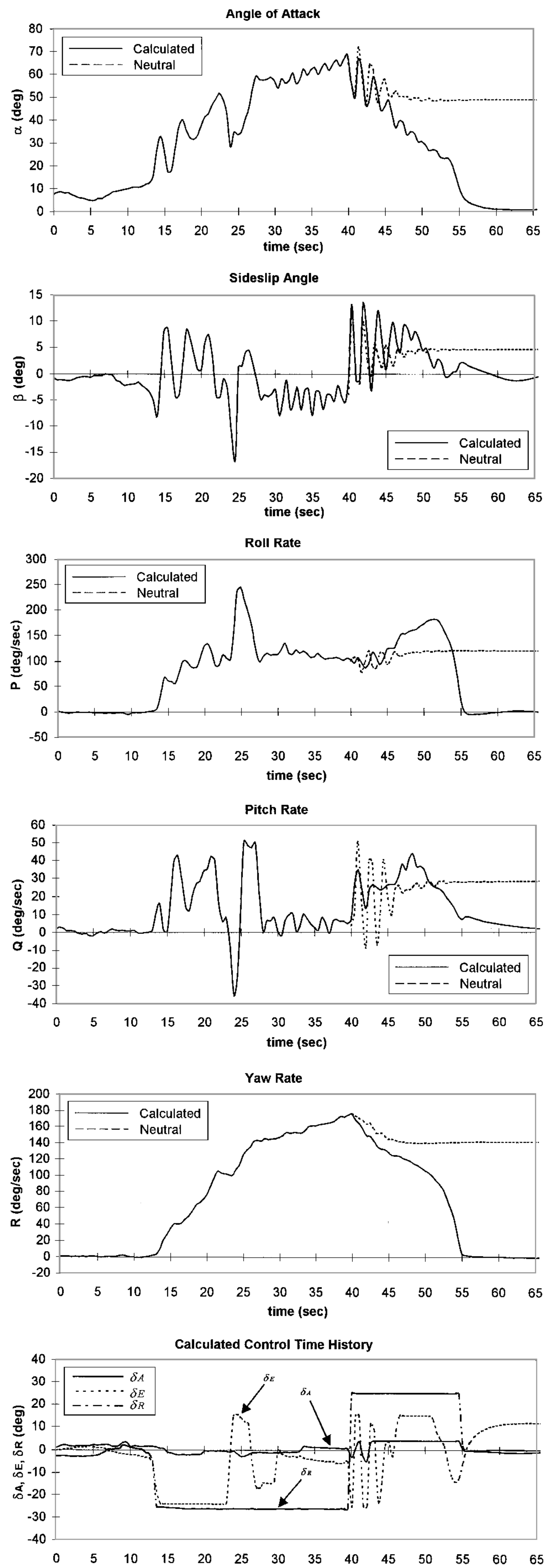

Fig. 7 Automatic recovery (——) and neutral controls (initiated at 40 s).

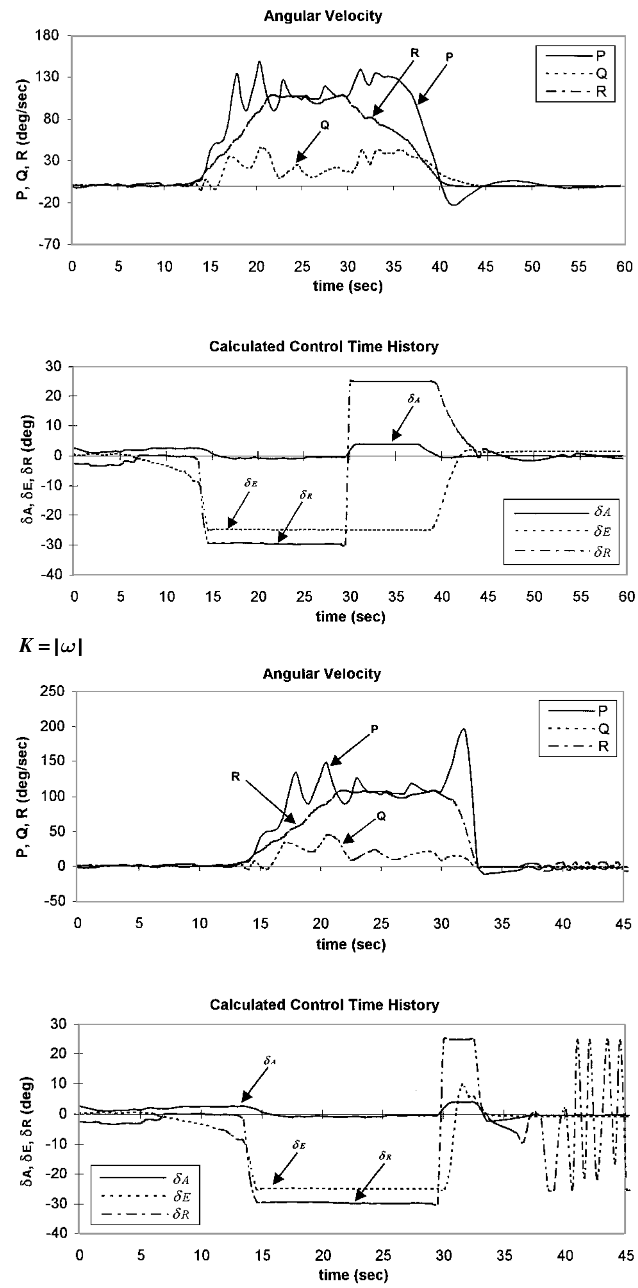

Fig. 8 Effect of *K*.

analogous to what one encounters in classical control design. It is depicted in Fig. 8.

Conclusions

A scheme for angular momentum control with the purpose of suppressing rotation in nonlinear motion is presented. Simulated results compared to spin flight test data show the validity of this approach, which computes control inputs similar to the ones recorded during flight tests. After the rotation is stopped or reduced, the recovery of aerodynamic angles, dynamic pressure, and attitude will then become a much easier task for the pilot.

Application of the suggested approach in this paper requires measurements of the angular velocity. Currently available inexpensive and lightweight gyros, e.g., turning fork and fiber optic, can be used in the proposed system to achieve this objective.

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